where g=gravity=32, l=length to center of oscillation=$\frac{2}{3}a$, h=height of zero velocity=$2l$ without a sensible error, t=time, θ=angle counted from the down point. Reducing, we have,

$$t=\tfrac{1}{2}\sqrt{\left(\frac{l}{g}\right)}\int_{\frac{1}{4}\pi}^{\pi-\phi}\frac{d\theta}{\cos\frac{1}{2}\theta}=\sqrt{\left(\frac{l}{g}\right)}\Big[\log\tan(\tfrac{1}{4}\pi+\tfrac{1}{4}\theta)\Big]_{\frac{1}{4}\pi}^{\pi-\phi}$$

$$=\sqrt{\left(\frac{l}{g}\right)}\log\frac{\cot\frac{1}{4}\phi}{\tan\frac{3}{8}\pi}=\sqrt{\left(\frac{l}{g}\right)}(8.647491).$$

But $l=\frac{2}{3}$ of 100. $\therefore t=\sqrt{(\frac{100}{48})}(8.647491)=12.481$ seconds.

AVERAGE AND PROBABILITY.

139. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Four points are taken at random on the surface of a given sphere; find the average volume of the tetrahedron formed by the planes passing through the points taken three and three.

No solution has been received.

140. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Obtain the average area of a triangle formed by a tangent to the four-cusped hypocycloid and the co-ordinate axes.

Solution by J. E. SANDERS, Hackney, Ohio.

The curve's equation is $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ The length of the perpendicular from the origin on the tangent is

$$\frac{xdx-ydy}{\sqrt{(dx^2+dy^2)}}=\frac{x^{\frac{2}{3}}+y^{\frac{2}{3}}}{\sqrt{(y^{-\frac{2}{3}}+x^{-\frac{2}{3}})}}=\sqrt[3]{(axy)}.$$

The equation to the tangent is $x'/x^{\frac{1}{3}}+y'/y^{\frac{1}{3}}=a^{\frac{2}{3}}$, and its length between the axis$=a^{\frac{1}{3}}\sqrt{(x^{\frac{2}{3}}+y^{\frac{2}{3}})}=a$.

$\therefore$ The area of the triangle is $A=\frac{1}{2}a\sqrt[3]{(axy)}=\frac{1}{2}a^2\sin\theta\cos\theta$, if $x=a\cos^3\theta$, and $y=a\sin^3\theta$.

Then, if the tangent is through any point on the curve, the average area is

$$\triangle=\frac{1}{s}\int A ds.$$

But $ds=a^{\frac{1}{3}}x^{-\frac{1}{3}}dx=3a\sin\theta\cos\theta\, d\theta$ and $s=\frac{3}{2}a$.

$\therefore \triangle=a^2\int_0^{\frac{1}{2}\pi}\sin^2\theta\cos^2\theta\, d\theta=\frac{1}{16}\pi a^2$, or, if the tangent is to be a random line, the average area is